Some Applications of Multivariate Multilevel Models in the Context of ESM/EMA Studies

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Purpose

- multivariate models are for multiple outcomes
- some applications:
  - examine reliability (internal consistency) of multiple items measuring the same construct
  - estimate correlation between two outcomes (and how correlation differs across groups)
  - test whether a predictor has more/less influence on outcome 1 vs outcome 2

2

Reliability of ESM/EMA Data

- examine reliability (internal consistency) of multiple items measuring the same construct
- use measurement of mood (positive and negative affect) as an illustrate example

3

Illustrative Data

- simulated dataset for an ESM/EMA study
- 10 semi-random beeps in 90-min intervals between 7:30am and 10:30pm for 4 days
- n = 100 subjects (52 females, 48 males)
- 3 items for PA, 4 items for NA (1-7 scale)

4

Reliability of PA and NA

- cannot examine reliability by examining consistency of measurements over time
- but can examine consistency of multiple items measuring the same construct
- analysis approach: multilevel factor analysis

5

Steps

1. restructure data into ‘very-long’ format
2. fit multivariate multilevel model
3. …
4. **PROFIT!**

6

id	beep	pa1	pa2	pa3	na1	…
1	1	2	7	4	4	…
1	2	1	6	4	6	…
…	…	…	…	…	…	…

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id	beep	item	itemnum	y
1	1	pa1	1	2
1	1	pa2	2	7
1	1	pa3	3	4
1	1	na1	4	4
…	…	…	…	…
1	2	pa1	1	1
1	2	pa2	2	6
1	2	pa3	3	4
1	2	na1	4	6
…	…	…	…	…

Multivariate Multilevel Model

$$y_{ijk} = \mu_k + u_{ik} + u_{ijk}$$

y_{ijk} = response of person i at beep j for item k

μ_k = mean of item k (averaged over persons and beeps)

u_{ik} = random effects for items at the person level

u_{ijk} = random effects for items at the beep level

$$\text{Var}\begin{bmatrix} u_{i1} \\ u_{i2} \\ u_{i3} \\ \ldots \end{bmatrix} = \begin{bmatrix} \tau_1^2 & \rho_{12}\tau_1\tau_2 & \rho_{13}\tau_1\tau_3 & \ldots \\ \ldots & \tau_2^2 & \rho_{23}\tau_2\tau_3 & \ldots \\ \ldots & \ldots & \tau_3^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix} \quad \text{Var}\begin{bmatrix} u_{ij1} \\ u_{ij2} \\ u_{ij3} \\ \ldots \end{bmatrix} = \begin{bmatrix} \nu_1^2 & \phi_{12}\nu_1\nu_2 & \phi_{13}\nu_1\nu_3 & \ldots \\ \ldots & \nu_2^2 & \phi_{23}\nu_2\nu_3 & \ldots \\ \ldots & \ldots & \nu_3^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$

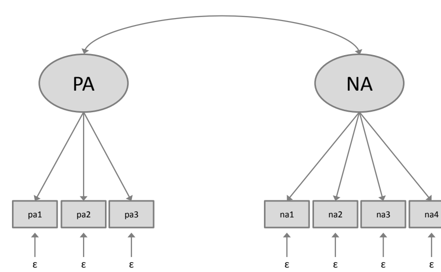

```
library(nlme)
res <- lme(y ~ item - 1,
          random = ~ item - 1 | id,
          weights = varIdent(form = ~ 1 | item),
          correlation = corSymm(form = ~ itemnum | id/beep),
          data = dat, na.action = na.omit,
          control = list(msVerbose=TRUE, maxIter=10000,
                         msMaxIter=10000, msMaxEval=10000))
summary(res)
```

Notes:

- `item` must be a factor/character variable (use `factor()` if needed)
- beep is a counter from 1 to the maximum number of possible beeps
- `na.action = na.omit` only needed if there are missings
- the `control` part is optional, but may be needed to get convergence
- model fitting can take some time

Two-Factor CFA Model

based on estimated correlations at the person and beep level, fit two-factor CFA model



Estimate Reliability

then estimate reliability from CFA model using McDonald's omega

$$\omega = \frac{\left(\sum_{i=1}^{p} \hat{\lambda}_i\right)^2}{\left(\sum_{i=1}^{p} \hat{\lambda}_i\right)^2 + \sum_{i=1}^{p} \hat{\sigma}_i^2}$$

p = number of items loading on factor

$\hat{\lambda}_i$ = loading of item i

$\hat{\sigma}_i^2$ = error variance of item i

Example Data and Code

- example dataset and code on GitHub

https://github.com/wviechtb/multivariate_multilevel_models

Estimate Correlation

- estimate correlation between two outcomes (and how correlation differs across groups)
- some approach this by making one outcome the 'outcome' and the other the 'predictor' (with some kind of standardization)
- but BOTH variables are outcomes, so we should treat them as such

id	sex	beep	outcome	outnum	y
1	female	1	PA	1	4.33
1	female	1	NA	2	2.50
1	female	2	PA	1	3.67
1	female	2	NA	2	3.75
…	…	…	…	…	…
1	female	38	PA	1	3.67
1	female	38	NA	2	2.75
1	female	40	PA	1	2.67
1	female	40	NA	2	3.50
…	…	…	…	…	…


```
library(nlme)
res <- lme(y ~ outcome - 1,
           random = ~ outcome - 1 | id,
           weights = varIdent(form = ~ 1 | outcome),
           correlation = corSymm(form = ~ outnum | id/beep),
           data = dat, na.action = na.omit,
           control = list(msVerbose=TRUE, maxIter=10000,
                          msMaxIter=10000, msMaxEval=10000))
summary(res)
```

Notes:

- `outcome` must be a factor/character variable (use `factor()` if needed)
- `beep` is a counter from 1 to the maximum number of possible beeps
- `na.action = na.omit` only needed if there are missings
- the `control` part is optional, but may be needed to get convergence
- model fitting can take some time

Inference (CIs/Testing)

- get CIs for variance/correlation components with `intervals(res)`
- can also compute CI by hand using the Fisher transformation (see wikipedia)
  - n for person level correlation: number of subjects
  - n for beep level correlation: total number of beeps (careful: this is # of rows in long format / 2)
  - not the same as `intervals()` but very close

Inference (CIs/Testing)

- to compare correlation across groups:
  - fit model within each group
  - then test correlations against each other using Fisher transformation
- testing for an association with a continuous predictor is tricky (need to use model that allows for predictors of variance/correlation components)

Differential Influence Predictors

- test whether a predictor has more/less influence on outcome 1 vs outcome 2
- inadequate (but often used) approach: fit out1 ~ pred and out2 ~ pred and see if one is significant and the other is not
- also cannot easily test coefficients against each due to dependence
- using multivariate multilevel model is better

id	sex	beep	outcome	outnum	y
1	female	1	PA	1	4.33
1	female	1	NA	2	2.50
1	female	2	PA	1	3.67
1	female	2	NA	2	3.75
…	…	…	…	…	…
1	female	38	PA	1	3.67
1	female	38	NA	2	2.75
1	female	40	PA	1	2.67
1	female	40	NA	2	3.50
…	…	…	…	…	…

Differential Influence Predictors

- fit model with outcome, predictor, and the outcome x predictor interaction
- test of interaction is test of differential influence
- this accounts properly for dependence

Reference

- useful references for these types of models:

  - Hox, J. J. (2010). Multilevel analysis: Techniques and applications (2nd ed.). New York: Routledge.
  - Goldstein, H. (2011). Multilevel statistical models (4th ed.). Chichester, UK: Wiley.